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CHARACTERISTICS OF A PLANE TURBULENT JET IN A BOUNDED DRIFTING FLOW

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A method is proposed for computing the characteristics of a plane turbulent jet escaping at a right angle to a flow constrained by channel walls. Results are presented of an experimental investigation of such jets and their comparison with design data.

A large number of papers is devoted to the theoretical and experimental investigation of jets in drifting flow. However, a systematic experimental investigation of the influence of the boundedness of the flow on the plane jet propagation characteristics has, visibly, not been performed. In the known papers [1-3] the investigations were carried out just in channels with specific geometry and, consequently, mainly just the jet trajectories were determined in the experiment. A theoretical solution of the problem of determining the characteristics of plane turbulent jets escaping at an angle to the flow constrained by channel walls does not exist in the literature, insofar as we know.

A method is proposed below for the computation of such jets on the basis of the solutions of problems on a plane jet in an unbounded drifting flow [4-7] and on the rarefaction in the reverse flow zone behind a plane jet in a constrained drifting flow [8]. Moreover, results of an experimental investigation of the influence of structural and dynamical parameters on characteristics of a plane jet are elucidated.

Sergo Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 6, pp. 904-911, June, 1985. Original article submitted April 16, 1984.

0022-0841/85/4806-0649$09.50

A solution of the problem of a plane jet in an unbounded drifting stream [4-6] is based on two statements: 1) the boundary layer equations are valid in curvilinear coordinates coupled to the jet axis, and 2) the boundary conditions can be given approximately by considering transverse stream flow around the jet as flow around a curvilinear wall. The magnitude of the refraction, determined theoretically in [9] in the reverse flow zone as well as the jet trajectory and the stream velocity at its forward boundary were utilized in [7] in determining the jet characteristics. In connection with the fact that the reverse flow zone is closed, it was here considered that the "displacement body" formed by the jet has the shape of an ellipse.

The fundamental ideas of the solution [4-7] can evidently be used even in the case of jet escape into a drifting stream constrained by channel walls. Even the jet boundary layer equations remain. However, as is shown in [8], the presence of a stream constraint changes the magnitude of the refraction substantially in the reverse flow zone and also the jet trajectory and, therefore, induces serious changes in the conditions on its forward boundary. Therefore, all the parameters for a plane jet in a bounded drifting stream can be determined from formulas in [7], but changes in the trajectory, the rarefaction, and the flow conditions around the jet should here be taken into account.

The velocity on the axis u_m, the forward and rear jet boundaries, and the pressure distribution were determined by an integral method in [7] under the assumption of similarity of the velocity profiles in the forward and rear parts of the jet. Used here as the equation to determine the axial velocity is an integral relationship between the pulses in the rear part of the jet which is obtained by integrating the equation of motion across the jet under the assumption that the velocity in the reverse flow zone is zero:

$$\frac{d}{dx}\int_{\delta_2}^{0}\left(u^2+\frac{p}{\rho}\right)dy=0. \tag{1}$$

The magnitude of the pressure p/ρ is here determined from the transverse equilibrium condition (the second equation of motion)

$$\frac{\partial p}{\partial y}=\frac{1}{\rho}\frac{u^2}{R}, \tag{2}$$

where R is the radius of curvature of the jet axis from which

$$\frac{p}{\rho}=-\frac{u_m^2}{R}\delta_2\left(0.316-\eta_2+\frac{8}{5}\eta_2^{5/2}-\frac{3}{2}\eta_2^4+\frac{8}{11}\eta_2^{11/2}-\frac{1}{7}\eta_2^7\right) \tag{3}$$

is obtained for the description of the velocity by the Schlichting formula. Here δ_2 is the ordinate of the rear jet boundary, $\eta_2 = y/\delta_2$.

By using (3) a formula for the velocity distribution along the jet axis is obtained from (1) after manipulation

$$u_m^2=-\frac{3.16K_2}{\delta_2(1-0.209\delta_2/R)},$$
$$K_2=b_{02}\int_0^1\left(u^2+\frac{p}{\rho}\right)_0 dy=b_{02}u_0^2\left(1+\frac{1+\Delta p}{4}\frac{V_\infty^2}{u_0^2}\right). \tag{4}$$

The relationship between the ordinates δ_1 and δ_2 of the jet boundaries can be found from the condition of equality of the derivatives of the tangential stress $\partial\tau/\partial y$ on the axis, written by using the velocity profiles for the jet forward and rear parts:

$$\frac{u-u_{\delta i}}{u_m-u_{\delta i}}=\left[1-\left(\frac{y}{\delta_i}\right)^{3/2}\right]^2=f(\eta_i) \tag{5}$$

($i = 1$ for the jet forward part, $i = 2$ for the rear, $u_{\delta i} = u_\delta$ for $i = 1$, and $u_{\delta i} = 0$ for $i = 2$).

Taking account of (5) in conformity with the Prandtl formula, the expressions for the tangential stresses have the form

$$\tau_1 = -\frac{\rho l^2}{\delta_1^2}(u_m - u_\delta)^2 f'^2(\eta_1), \quad \tau_2 = \frac{\rho l^2}{\delta_2^2} u_m^2 f'^2(\eta_2). \tag{6}$$

We consider the mixing path l constant in the jet section and identical for the forward and rear parts. Differentiating the expression for the tangential stress (6) with respect to y, we find for y = 0

$$\left(\frac{\partial \tau_1}{\partial y}\right)_{y=0} = -9\frac{\rho l^2}{\delta_1^3}(u_m - u_\delta)^2, \quad \left(\frac{\partial \tau_2}{\partial y}\right)_{y=0} = 9\frac{\rho l^2}{\delta_2^3} u_m^2. \tag{7}$$

Equating the right sides of (7), we obtain the relationship between the ordinates for the jet forward and rear boundaries

$$\delta_1 = -\delta_2\left(1 - \frac{u_\delta}{u_m}\right)^{2/3}. \tag{8}$$

Let us note that the same relationship is obtained in [4] by using the comparison of formulas for the velocity profiles in the jet forward and rear parts as found by the method of polynomial representation of the tangential stress profiles while merging these profiles on the jet axis. In this case the velocity profile was obtained different from the Schlichting profile (5).

To determine the ordinate δ_2 of the jet rear boundary, the "condition on the axis" [10] could be used, i.e., the equation of motion written for the jet axis

$$u_m u_m' = \left(-\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}\right)_{y=0}. \tag{9}$$

After some manipulation with (4) taken into account, we obtain

$$\delta_2'(x) = -18\beta^2\left(1 + 0.209\frac{\delta_2}{R}\right), \tag{10}$$

where $\beta = l/\delta_2$ is an empirical constant.

The magnitude of the velocity on the jet boundary in (8) and the jet trajectory were determined from the transverse potential flow around a cascade of elliptic cylinders, as is mentioned in [8], obtained by using [11]. The streamlines, one of which (and its continuation) agrees with the cylinder axis while the other is the axis of symmetry between two adjacent elliptical cylinders, play the part of walls in this case. The separating streamline, the line of constant discharge that separates the initial jet mass from the mass attached to the jet, was selected as the "displacement body" contour formed by the jet. In contrast to the displacement body contour in [9] this line is impermeable. Then the jet trajectory was determined as the geometric locus of points standing off from the "displacement body" boundary by the magnitude of the ordinate of the separating stream line y_s in the coordinates coupled to the jet axis. This ordinate is determined from the equation

$$\int_0^{y_s} u\,dy = u_0 b_{01}. \tag{11}$$

FORTRAN programs for computation on electronic computers were produced to calculate the jet trajectories with rarefaction behind the jet taken into account, which depends substantially on the degree of flow constraint (see [8]) as well as the flow and jet characteristics. Let us note that because the jet velocity changes slightly near the axis, for approximate computations the quantity y_s can even be determined without using an electronic computer by setting $u = u_0$ at the width y_s in the initial section and $u = u_m$ in the main section. We then find from (11)

$$\frac{y_p}{b_{01}} = \frac{u_0}{u_m}. \tag{12}$$

To compute the trajectory of a jet being propagated in a transverse stream, graphs for the quantities m and a obtained by using an electronic computer with the above-mentioned program can be used and are presented in Fig. 1. Then the coordinates are easily computed for the jet axis, interrelated approximately by the formula for an ellipse

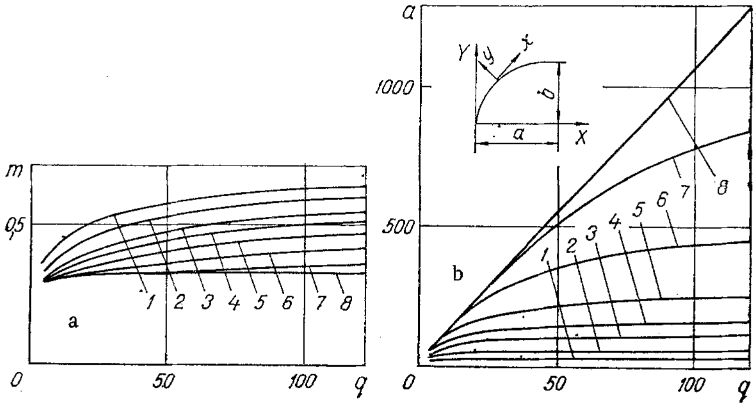

Fig. 1. Parameters governing the shape of the "displacement body" formed by a plane jet in a constrained transverse flow: a: 1) H/b_0 = 25; 2) 50; 3) 100; 4) 150; 5) 250; 6) 500; 7) 1250; 8) 9000 (∞); b: 1) H/b_0 = 25; 2) 50; 3) 100; 4) 150; 5) 250; 6) 500; 7) 1250; 8) 5000 (∞).

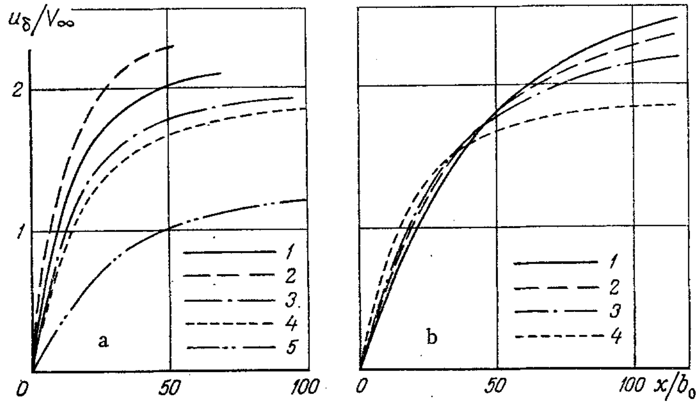

Fig. 2. Velocity on the boundary of a jet in a constrained transverse flow: a: 1) H/b_0 = 54.4, q = 27.4; 2) 39.0, 27.2; 3) 80, 28.7; 4) 100, 31.7; 5) ∞, 31.1; b: H/b_0 = 100: 1) q = 97.6; 2) 79.3; 3) 61.9; 4) 31.7.

$$\frac{(a-X)^2}{a^2}+\frac{Y^2}{b^2}=0, \tag{13}$$

while the distance along the jet trajectory to a given section and the radius of curvature at each point along the jet trajectory are determined from the formulas

$$x=\int_0^Y\left[1+\left(\frac{dX}{dY}\right)^2\right]^{1/2}dY,\quad R=\frac{a^2}{b}\left[1-\frac{c^2}{a^4}(a-X)^2\right]^{3/2},\ c^2=a^2-b^2 \tag{14}$$

[the coordinate systems taken that are coupled to the jet source and its trajectories (X, Y) and (x, y) are shown in Fig. 1b].

The change in velocity on a jet boundary along its trajectory is presented in Fig. 2 as computed for different values of q and H/b_0 = 100 (Fig. 2a), and for nearby values of q and different quantities H/b_0. It is seen that both parameters exert substantial influence on the quantity u_δ/V_∞.

Computation of the parameters of the main part of a plane jet being propagated in a drifting stream can be carried out from the transition section in which we consider $u_m = u_0$ and dp/dx small. Then, neglecting the quantity δ_2/R in a first approximation, we find from (4)

$$\delta_{2t}=-3.16b_{02}. \tag{15}$$

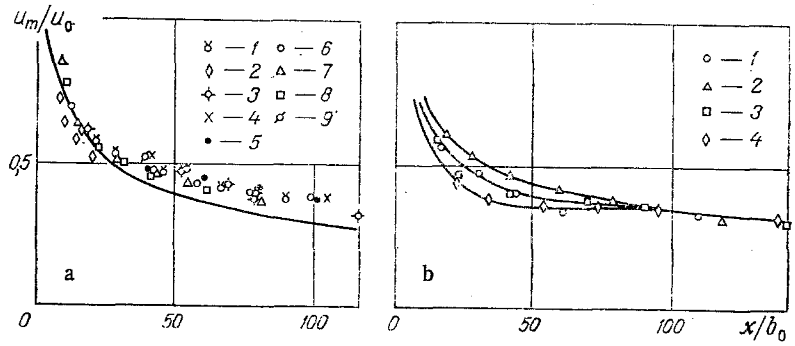

Fig. 3. Axial velocity distribution: a: 1) H/b_0 = 100, q = 18.8; 2) 40, 19.3; 3) 80, 37.6; 4) ∞, 31.2; 5) 100, 31.7; 6) 80, 28.2; 7) 54.4, 28.2; 8) 39, 27.2; 9) ∞, 21.2; b: 1) α_0 = 60°, q = 37.3; 2) 90°, 37.6; 3) 120°, 37.6; 4) 150°, 39.1; H/b_0 = 80.

From the value of δ_{2t} found in this manner, we find the abscissa of the transition section measured from the nozzle exit by roughly considering approximately that the jet boundary up to this section varies linearly according to the formula

$$\delta_2 = -18\beta^2 x. \tag{16}$$

Then

$$x_t = \frac{3,16 b_{02}}{18\beta^2}. \tag{17}$$

The abscissa of the transition section can be found more exactly by solving the problem of the initial section of such a jet and continuing the jet boundary linearly from the end of the initial section to the transition section [11]:

$$x_t = x_i - \frac{\delta_{2t} - \delta_{2i}}{18\beta^2}. \tag{18}$$

From the value found for x_t there are obtained u_δ/V_∞, R, and the coordinates X_t and Y_t, where it is taken into account that x_t is the arclength from the ellipse vertex to the given point (transition section) determined by means of (14), while X_t and Y_t are related by the formula for the ellipse (13). By means of the values of x_t and δ_{2t} determined in this manner, and, therefore, u_δ, it is possible to find δ_{1t} by means of (8). Furthermore, by giving new values of $Y > Y_t$, the corresponding values of X, x, u_δ, and R can be determined. Then by integrating (10) under the condition $\delta_2 = \delta_{2t}$ for $x = x_t$, the magnitudes of the velocity on the axis u_m and the ordinates of the outer (in front of the stream) boundary of the jet δ_1 can be found by means of (4) and (8).

To study the influence of the structural and dynamical parameters on the flow characteristics in the jet being developed in a constrained drifting stream, and to approve the method elucidated above for computing these characteristics in addition to the study of the pressure distribution on the channel walls and in the reverse flow zone behind the jet (see [8]), an experimental investigation was conducted for the velocity and pressure profiles in transverse sections of the jet for different degrees of drifting stream constraint H/b_0 and different ratios between the jet velocity head and the stream q. The experimental apparatus produced to study development of a plane jet and a system of circular jets in a constrained drifting stream is described in [8].

The velocity heads, the angles of the velocity vector, and the static pressure distributions in sections perpendicular to the jet axis found first as the maximal velocity line were measured in the experimental investigation of the main section of the jet by using a four-channel combined nozzle. The jet boundaries and the change in axial velocity along the jet trajectory were determined from these data. Processing the velocity profiles in dimensionless coordinates showed that these profiles are similar in the whole range of variation investigated for the degree of flow constraint (H/b_0 = 20-100 and H/b_0 = ∞) and the ratio between the jet velocity heads and the stream (q = 18-100).

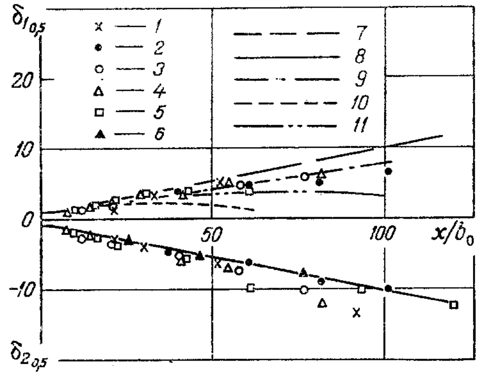

Fig. 4. Forward and rear jet boundaries in an unbounded and constrained drifting stream: 1-6) Experiment: 1) $H/b_0 = \infty$, $q = 31.2$; 2) 100, 31.7; 3) 80, 28.2; 4) 54.4, 29.2; 5) 39, 24.2; 6) submerged jet; 7-10) analysis: 7) submerged jet; 8) δ_2; 9) $H/b_0 = \infty$, $q = 31.2$; 10) 100, 31.7; 11) 100, 31.7 (with correction).

The investigation showed that the flow constraint has substantial influence on the location of the jet axis. As the degree of stream constraint increases (diminution of the ratio between the channel height and the slot width from which the jet is propelled H/b_0) the jet is deflected more strongly under the action of the drifting stream (see [8]).

The dependence of the axial velocity on the distance to the nozzle exit, determined along the axis, is represented in Fig. 3 as obtained for different values of H/b_0 and nearby values of q (Fig. 3a). It is seen that a change in H/b_0 has practically no influence on the change in axial velocity. The results of the axial velocity computations (curve) performed by means of (4) are also in agreement with this deduction. An analogous result is also obtained for changes in q. The magnitude of the angle of jet expulsion turns out to be the single parameter that noticeably influences the change in axial velocity along the jet trajectory. The change in the axial velocity in the channel with relative height $H/b_0 = 80$ is is shown in Fig. 3b for jet expulsion angles varying between 60 and 150°. It is seen that the curves stratify in the initial domain of the main section, and for expulsion angles different from 90° the drop in the axial velocity occurs more intensively. The velocity drop for the expulsion angles 120 and 150° is especially noticeable. Far from the jet source the difference in the magnitudes of the axial velocity for different angles α practically vanishes.

The forward and rear jet boundaries, determined as the lines of half the excess velocities and obtained for different quantities H/b_0 and nearby q are shown in Fig. 4. The computed rear jet boundary which turns out to be practically coincident with the boundary of a submerged jet is displayed by the solid line. The boundary of the submerged jet at half the velocity is superposed by the long dashes in the upper part of the figure. It is seen that the jet rear boundary found from test is close to the submerged jet boundary although a certain tendency to an increase in the degree of jet expansion is also traced at large distances from the jet source as the degree of stream constraint increases. As regards the jet forward boundary, it can be said with respect to it that there is just qualitative agreement between the computed and experimental data since the degree of jet expansion in its forward section is less in both computation and experiment than the degree of submerged jet expansion, and, therefore, the degree of expansion of the rear part of the jet to the stream, but the computed values of the ordinates of the forward jet boundaries are explicitly reduced.

Comparison with test showed that the computed magnitude of the velocity on the boundary of the "displacement body" formed by the jet is somewhat higher than is obtained in experiment while the magnitude of the axial velocity is somewhat below the test value. If an appropriate empirical correction is introduced in these quantities, then the relationship between the ordinates of the forward and rear jet boundaries turns out to correspond to (8).

It is possible to try to compensate the exaggeration in the computation of the quantity u_δ and the reduction in u_m as compared with the test data by introducing the magnitude of the initial mean velocity u_0 in place of u_m in (8), and to consider the forward part of the jet as a jet in a stream with variable u_δ/u_0. The result of computing the jet forward boundary by such a corrected formula is shown by curve 11 in Fig. 4 for $H/b_0 = 100$ and $q = 31.7$. It is seen that the agreement with test upon insertion of such a correction is completely satisfactory.

NOTATION

a, b, values of the major and minor ellipse semiaxes; $b_0 = b_{01} + b_{02}$, slot width out of which the jet streams; $c^2 = a^2 - b^2$; H, channel height; K_2, kinematic impulse of the rear part of the jet to the flow; l, mixing path; $m = b/a$; p, pressure; p_∞, pressure far in front of the jet; p_0, pressure in the reverse flow zone behind the jet; q, ratio of the jet and stream velocity heads; R, radius of curvature; u, longitudinal velocity component; u_0, u_m, mean jet escape and axial velocity; u_δ, velocity at the jet forward boundary; V_∞, drifting flow velocity; x, y, curvilinear coordinates; x, along the jet axis and y is perpendicular to x; X, Y, coordinates coupled to the nozzle exit; X, along the drifting stream, and Y is perpendicular to X; x_i, x_t, abscissas of initial and transition section ends; y_s ordinate of the separating streamline; β, an empirical constant; δ_1, δ_2, ordinates of the jet forward and rear boundaries; Δp, relative rarefaction in the reverse flow zone behind the jet; $\eta_i = y/\delta_i$ (i = 1 for the forward jet part to the flow, and i = 2 for the rear part); ρ, density; τ_1, τ_2, tangential stresses in the forward and rear parts of the jet to the flow.

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